Report, NASA Grant NSG-1181, Howard University, Dept. of Mechanical Engineering, Washington, D.C., May 1976.

[3]Bainum, P.M., Fuechsel, P.G., and Mackison, D.L., "Motion and Stability of a Dual-Spin Satellite with Nutation Damping," *Journal of Spacecraft and Rockets,* Vol. 7, June 1970, pp. 690-696.

Minimization of Temperature Distortion in Thermocouple Cavities

Ching Jen Chen* and Peter Li†
The University of Iowa, Iowa City, Iowa

Introduction

A DIRECT measurement of transient surface temperature and heat flux is often difficult. For example, a surface involves two modes of heat transfer, say radiative and convective heat transfer. In this case, if the measuring probe has a different radiative property from that of the surface, erroneous measurements will result. Therefore, indirect estimation by inverting the temperature history inside the heat conducting solid as measured by a thermocouple is often used for prediction of the surface temperature and heat flux. Beck,[1] Herring and Parker,[2] Frank,[3] Imber and Khan,[4] Stolz,[5] and Chen and Thomsen[6] have developed inversion solutions for this purpose. Since all of these solutions assumed that the cavity drilled into the solid does not distort the true temperature distribution, it is, therefore, important that the temperature measurement by an interior probe be accurate and involve little distortion or error. From studies made by Chen and Li[7] and Beck,[8] it was found that with a proper combination of the thermocouple cavity diameter, cavity depth, and the thermocouple material, the magnitude of the distortion of the temperature field with respect to space or time can be minimized. In this Note we study the optimum combination of geometrical parameters and material properties to eliminate the temperature distortion.

Analysis

In the present study, we consider a disk depicted in Fig. 1, which has a thickness D and a cavity of diameter d drilled to a depth of ϵ distance from the heated surface. The heat flux Q is assumed to be constant and the upper surface of the disk is assumed to be insulated. A thermocouple of diameter d_t is welded onto the cavity base. The diameter of the disk is chosen to be $2D$ with the temperature distortion due to the thermocouple cavity assumed to be negligible at the edge. For this to be true Chen and Li[7] showed that the ratio of the cavity diameter to the disk diameter $d/2D$ should be smaller than 0.1. The portion of the cavity not filled by the thermocouple can be air or insulating material. The basic idea used to minimize or to eliminate the temperature distortion is based on a proper choice of the thermocouple size and the thermocouple material, which has a higher thermal conductivity than that of the disk, so as to conduct more heat away at the cavity base balancing the insulation effect of the insulator in the cavity.

Let X and Y be the respective coordinates along the heated disk surface and the axis of the cavity. The thermal conductivity is assumed to be constant. The governing equations for transient heat conduction in dimensionless form are

$$\frac{\partial\theta}{\partial\tau}=\frac{\alpha_i}{\alpha_1}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{1}{x}\frac{\partial\theta}{\partial x}+\frac{\partial^2\theta}{\partial y^2}\right),\quad i=1,2,3 \tag{1}$$

where $\tau=\alpha_1 t/D^2$ is the dimensionless time, $x=X/D$ the dimensionless radial coordinate, and $y=Y/D$ the dimensionless distance normal to the heated surface. The α_i are the thermal diffusivities with subscripts 1, 2, and 3 denoting the disk, the insulating material, and the thermocouple. The dimensionless temperature θ is defined as $T\kappa_1/QD$ where T is the temperature above the initial uniform temperature and κ_1 is the thermal conductivity of the disk.

The initial temperature of the disk is

$$\theta(x,y,0)=0 \tag{2}$$

The boundary conditions (Fig. 1) are:

$$y=0,\ \left.\frac{\partial\theta}{\partial y}\right|_{y=0}=-1 \qquad y=1,\ \left.\frac{\partial\theta}{\partial y}\right|_{y=0}=0 \tag{3}$$

$$x=1,\ \left.\frac{\partial\theta}{\partial x}\right|_{x=1}=0 \qquad x=0,\ \left.\frac{\partial\theta}{\partial x}\right|_{x=0}=0 \tag{4}$$

There are five parameters that can be varied for the present analysis. They are: 1) the dimensionless distance from the base of the cavity to the heated surface ϵ/D, 2) the size of the cavity $d/2D$, 3) the ratio of the thermocouple diameter to that of the cavity d_t/d, 4) thermal conductivity ratios κ_2/κ_1 and κ_3/κ_1 which come from the continuity of heat flux at interfaces, and 5) the ratio of the product of density and specific heat ρ_3c_3/ρ_1c_1.

Because of the complexity of the geometry and the multiplicity of materials, the finite element technique as discussed by Wilson and Nikel[10] is adapted. The present problem is subdivided into finite elements as required by the method. The cross section is subdivided into 121 finite elements with 12 dividing lines on both coordinates. Each element is defined by four nodal points where nodal points are denoted by intersections of the dividing lines. The solution at each node with respect to time is then obtained.

For numerical calculations three typical values of the distance from the heated surface to the base of the cavity ϵ/D are chosen. They are 0.02, 0.06, and 0.1. The cavity diameter is fixed at one tenth of the disk diameter. The thermocouple to

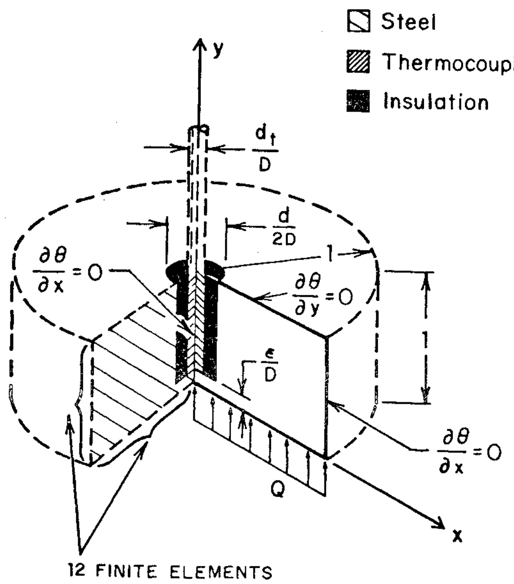

Fig. 1 Geometric representation of problem.

Received Oct. 20, 1976; revision received Feb. 28, 1977.

Index categories: Heat Conduction; Thermal Surface Properties; Rocket Engine Testing.

*Associate Professor, Dept. of Energy Engineering. Member AIAA.

†Research Assistant, Dept. of Energy Engineering.

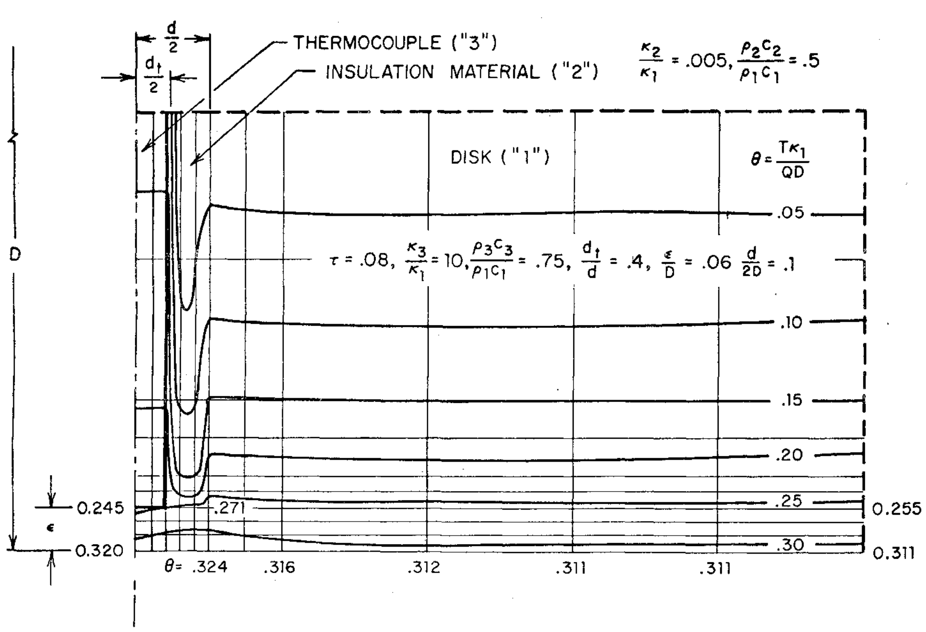

Fig. 2 Temperature distribution with thermocouple partially filling the cavity.

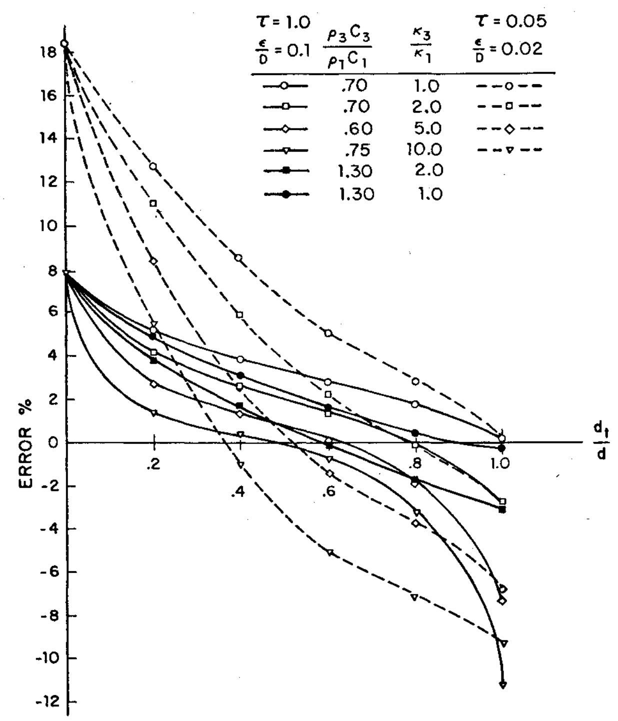

Fig. 3 Percentage temperature error as a function of d_t/d ratio.

cavity diameter ratio d_t/d is varied from 0 to 1.0. The range of the ratio of the thermal conductivity κ_3/κ_1 and the ratio of the product of the density and the specific heat ρ_3c_3/ρ_1c_1 for calculation are chosen to vary from 1 to 10 and 0.7 to 1.3, respectively, to cover the practical range. The corresponding values of $\rho_c c_2/\rho_1 c_1$ and κ_2/κ_1 for the insulation material are chosen to be 0.5 and 0.005, respectively, which are typical values for Teflon insulating material.

Results and Discussions

Numerical results of the calculations are presented in Figs. 2 and 3. The percentage error of temperature is defined as the distorted temperature divided by a reference temperature defined QD/κ_1. Figure 2 shows that by properly choosing the ratio of thermocouple diameter to that of the cavity, one may minimize the distortion of temperature response at the base of the cavity. In Fig. 2 the thermocouple diameter d_t is chosen to be 0.4 of the cavity diameter d with the ratio of thermal conductivity $\kappa_3/\kappa_1=10$, e.g., copper-steel combination, for the case $\epsilon/D=0.06$, $d/2D=0.1$, at the time $\tau=0.08$. The temperatures θ at the base of the cavity and at the heating surface are 0.245 and 0.320 which give errors of 1% and 0.9%, respectively when compared with the undistorted values of 0.255 and 0.311. To examine the details of the distorted temperature response at the base of the cavity the percentage error as a function of the ratio of the thermocouple diameter to that of the cavity is plotted in Fig. 3 for two cavity depths and times, and several ratios of thermal conductivity and ρc product. Figure 3 shows that when both ρ_3c_3/ρ_1c_1 and κ_3/κ_1 ratios are equal to or less than one, the error of the temperature response at the base of the cavity is always positive which indicates overheating. This is because both the thermal conductivity and the heat capacity of the thermocouple are less than that of the disk. Thus, no extra heat conduction can be achieved by the thermocouple to compensate for the blocking of the heat transfer by the insulation material in the cavity. On the other hand, if $\kappa_3/\kappa_1>1$ the temperature response error may vary from a positive value to some negative value depending on the ratio of d_t/d. Thus for $\kappa_3/\kappa_1>1$ a properly chosen combination of thermocouple and insulation material may minimize the error. For example, the in Fig. 3 a combination of $\kappa_3/\kappa_1=2$, $\rho_3c_3/\rho_1c_1=0.70$, $\epsilon/d=0.1$ and $d_t/d=0.8$ results in almost negligible error. This combination which appears to be optimum at $\tau=1.0$ is also found to be approximately valid for outer time periods. Therefore an optimum combination of parameters may be chosen for the entire transient period of an experiment. From Fig. 3 one can also see that the optimum ratio of d_t/d which gives zero temperature error decreased as the κ_3/κ_1 ratio increases. This implies that for the thermocouple with a larger thermal conductivity a smaller thermocouple diameter is sufficient to eliminate the temperature distortion. Figure 3 can thus be used as a guide for choosing an optimum combination of the size of thermocouple, the size of the cavity, and the properties of the thermocouple for a given application.

As mentioned earlier, the errors of the temperature response at the base of the cavity are all positive when $\kappa_3/\kappa_1 \leq 1$ and $\rho_3c_3/\rho_1c_1 \leq 1$. However we find that if the ρ_3c_3/ρ_1c_1 ratio is made large enough during the transient period the error of temperature response at the base of the cavity may indeed become negative even when $\kappa_3/\kappa_1 \leq 1$. This means that although the thermal conductivity of the thermocouple κ_3 is smaller than that of the disk material, with a larger heat capacitance $\rho_3c_3/\rho_1c_1>1$ the thermocouple is still capable of absorbing extra heat flux and hence eliminates the temperature distortion at the cavity base during the transient period. To illustrate this fact we examine Fig. 3 for the data $\kappa_3/\kappa_1 = 1$ and $\rho_3 c_3/\rho_1 c_1 = 1.3$. One sees that when $d_t/d = 1$ the temperature distortion can indeed be negative. Therefore, if d_t/d is chosen between 0.8 and 1 the error can be minimized. However one must keep in mind that the elimination of error by heat capacitance can work during the transient period only, for once steady-state conduction is established the heat capacity ρc will no longer have any affect, and overheating at the cavity base eventually will develop. This can be best seen from the governing equation, Eq. (1), where for steady state the unsteady term which contains the ρc product is zero and is not a parameter affecting the distortion.

Another important fact that should be mentioned is that, in general, the optimum choice of d_t/d ratio for a given κ_3/κ_1 and ρc ratio is rather insensitive to the variation of the ϵ/D ratio ranging from 0.02 to 0.1. This fact was already pointed out by Chen and Danh[9] in their experiment which demonstrated that the temperature distortion at the base of the cavity is more sensitive to the variation of the cavity diameter than the depth of the cavity drilled.

One disadvantage of invoking finite element analysis is that the result does not give a clear functional relation among the parameters involved. In an attempt to obtain a simple and approximate relation to relate the various parameters we note the following results: 1) the optimum d_t/d ratio for zero temperature distortion is a strong function of κ_3/κ_1 and ρc ratio but is relatively insensitive to the ϵ/D ratio, and 2) from theoretical reasoning the d_t/d ratio is independent of ρc ratio at steady state. A simple steady-state one-dimensional analysis in which the thermocouple and the insulation material in the cavity is made to conduct the same amount of heat that would be transferred without the cavity gives the relation

$$d_t/d = \sqrt{(\kappa_1 - \alpha_2)/(\kappa_3 - \kappa_2)} \qquad (5)$$

Using the above equation as a base we find that for transient heat conduction as calculated by the finite element method, Eq. (6) correlates very well with the optimum d_t/d ratio:

$$d_t/d = (\rho_3 c_3/\rho_1 c_1)^{0.3} \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \qquad (6)$$

Equation (6) gives a distortion error of no more than two percentage points. In practice, Eq. (6) may be used as a rule of thumb.

In summary, the temperature distortion caused by a cavity drilled into a disk to accommodate a thermocouple has been studied. The calculation is carried out for the case of constant heat flux. It is shown that the difference in temperature at the base of the cavity from that without a cavity can be eliminated by a properly chosen combination of the ratio of the thermocouple diameter to the cavity diameter, d_t/d, and the thermocouple material. The optimum ratio of d_t/d can be found from Fig. 3 or approximately from Eq. (6). As a rule, the thermocouple material must be chosen so that it has a higher thermal conductivity than that of the heat conducting solid. The cavity diameter should be as small as practically possible.

Acknowledgment

This work was supported by ARO research grant DAA-G29-76-G-0123.

References

[1]Beck, J. V., "Nonlinear Estimation Applied to the Nonlinear Inverse Heat Conduction Problem," *International Journal of Heat and Mass Transfer*, Vol. 13, 1970, pp. 703-716.

[2]Herring, C. D. and Parker, R., "Transient Response of an Instrinsic Thermocouple," *Journal of Heat Transfer, Transactions of the ASME*, Series G, Vol. 39, 1967, p. 146.

[3]Frank, I., "An Application of Least Square Method to the Solution of Inverse Problem of Heat Conduction," *Journal of Heat Transfer*, Vol. 85, 1963, pp. 378-379.

[4]Imber, M. and Khan, J., "Prediction of Transient Temperature Distributions with Embedded Thermocouple," *AIAA Journal*, Vol. 10, June 1972, pp. 784-789.

[5]Stolz, G., Jr., "Numerical Solutions to an Inverse Problem of Heat Conduction for Simple Shapes," *Journal of Heat Transfer*, Vol. 82, 1960, pp. 20-26.

[6]Chen, C. J. and Thomsen, D. M., "On Transient Cylindrical Surface Heat Flux Predicted from Interior Temperature Response," *AIAA Journal*, Vol. 13, May 1975, pp. 697-699.

[7]Chen, C. J. and Li, P., "Error Analysis of an Intrinsic Transient Heat Flux Sensor," ASME Paper 76-HT-62, *16th National Heat Transfer Conference*, St. Louis, Mo., Aug. 8-11, 1976.

[8]Beck, J. V., "Thermocouple Temperature Disturbances in Low Conductivity Materials," *Transactions of the ASME*, May 1962, pp. 124-131.

[9]Chen, C. J. and Danh, T. M., "Transient Temperature Distortion in a Slab Due to Thermocouple Cavity," *AIAA Journal*, Vol. 14, July 1976, pp. 979-981.

[10]Wilson, E. L. and Nickel, R. E., "Application of the Finite Element Method of Heat Conduction Analysis," *Journal of Nuclear Engineering and Design*, Vol. 4, 1966, pp. 276-286.

Nonlinear Flap-Lag-Axial Equations of a Rotating Beam

K. R. V. Kaza* and R. G. Kvaternik†
NASA Langley Research Center, Hampton, Va.

Introduction

THE literature dealing with the dynamics of rotating elastic bodies such as spin-stabilized satellites, helicopter rotor blades, and whirling beams has proliferated in the last decade. Representative papers treating the dynamic aspects of such structures are given in Refs. 1 to 11. Examining these references, one may identify essentially four approaches by which analysts have proceeded to establish either the linear or nonlinear governing equations of motion for the particular problem addressed. These are: the effective applied load artifice[2,3] in combination with a variational principle; the use of Newton's second law, written as D'Alembert's principle, applied to the deformed configuration;[1,7,10] a variational approach in which nonlinear strain-displacement relations and a first-degree displacement field are used;[5,9,10] and the method introduced by Vigneron[11] for deriving the linear flap-lag equations of a rotating beam. While these approaches complement one another, their application in some of the cited references reveals confusion regarding several aspects of the development of both the linear and nonlinear equations of motion of a rotating blade or beam. The confusion centers about: whether the geometric nonlinear theory of elasticity has been (or must be) used in deriving the equations of motion; whether Houbolt and Brooks[1] have considered geometric nonlinearity and thus have the necessary ingredients for extending their development to obtain the second-degree nonlinear equations; whether or not foreshortening must be considered explicitly by including it in the assumed axial displacement field; the use of the inex-

Received Dec. 10, 1976; revision received March 29, 1977.
Index categories: Structural Dynamics; Propeller and Rotor Systems.

*George Washington University-NASA Postdoctoral Research Associate, Aeroelasticity Branch. Now at NASA-Lewis Research Center, Cleveland, Ohio, (Senior Research Associate, University of Toledo, Toledo, Ohio). Member AIAA.
†Aeronautical Research Scientist, Aeroelasticity Branch. Member AIAA.